LOYOLA COLLEGE (AUTONOMOUS), CHENNAI- 6000034

M.Sc. DEGREE EXAMINATION- **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2015

MT 1817 - ORDINARY DIFFERENTIAL EQUATIONS

Date : 07/11/2015

Time : 01:00-04:00 Dept. No. MAX: 100 MARKS

Answer all questions. Each question carries 20 marks.

1. (a) Let $x_p(t)$ be any particular solution of $L[x(t)] = d(t)$ and $x_h(t)$ be the general solution of $L[x(t)] = 0$. Show that $x(t) = x_p(t) + x_h(t)$ is the general solution of $L[x(t)] = d(t)$. **(5)**

(OR)

(b) State and prove Abel's formula. **(5)**

(c) Explain the method of variation of parameters. **(15)**

(OR)

(d) If Wronskian of two functions $x_1(t)$ and $x_2(t)$ on I is non-zero for at least one point on the interval I, prove that $x_1(t)$ and $x_2(t)$ are linearly independent on I. Check whether the given sets of functions are linearly independent. (i) $\sin x$, $\sin 2x, \sin 3x$ on $I = [0,2\pi]$ (ii) $1 + x$, $x^2 + x, 2x^2 - x - 3$ and (iii) $1, x, x^2, \dots, x^n$ **(15)**

2. (a) Find the indicial equation of $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$. **(5)**

(OR)

(b) State and prove Rodrigure's formula. **(5)**

(c) Show that $\frac{1}{\sqrt{1-2tx+t^2}} = \sum_{l=0}^{\infty} t^l P_l(x)$ if $|t| < 1$ and $|x| \leq 1$. **(15)**

(OR)

(d) Solve $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + ky = 0$ by Frobenius method. **(15)**

3. (a) Obtain the generating function of Bessel's function. **(5)**

(OR)

(b) Prove that $J_n'(x) = \frac{n}{x}J_n(x) - J_{n+1}(x)$. **(5)**

(c) State and prove the integral representation of Bessel's function. **(15)**

(OR)

(d) Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$. **(15)**

4. (a) Is there any general criterion to ensure the Lipschitz condition? Justify. **(5)**

(OR)

(b) For distinct parameters λ and μ, let x and y be the corresponding solutions of the Strum-Liouville problem such that $[pW(x,y)]_A^B = 0$. Prove that $\int_A^B r(s)x(s)y(s)ds = 0$. **(5)**

(c) Prove that $x(t)$ is a solution of $L[x(t)] + f(t) = 0, a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$. **(15)**

(OR)

(d) State and prove Picard's theorem for boundary value problem. **(15)**

5. (a) Explain stable solution with an example. **(5)**

(OR)

(b) Prove that the system $x_1' = -3x_1 + kx_2, x_2' = -2x_1 - 4x_2$ is asymptotically stable for all x. **(5)**

(c) Explain the stability of $x' = Ax$ by Lyapunov's method. **(15)**

(OR)

(d) State and prove the fundamental theorems on the stability of non-autonomous systems. **(15)**

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